Multiline Insurance:

Bundling Risks to Reduce Moral Hazard

Christian Laux

J.W. Goethe-University Frankfurt

laux@finance.uni-frankfurt.de

February 2004

Preliminary and Incomplete

Abstract

I analyze the role of bundling risks to be covered by one multiline integrated risk management policy in the presence of moral hazard. The structure of the optimal multiline policy depends on a firm's risk-management objective. It is shown that—subject to retaining incentives to invest in risk reduction—the firm might be able to reduce the maximum retained loss by choosing a multiline policy with a common aggregate deductible. In contrast, a common aggregate policy limit is optimal if the objective is to minimize the expected retained loss. Therefore, multiline insurance might allow an improved trade-off between risk transfer and incentives.

JEL Classification: G22, D82

Keywords: multiline insurance, integrated risk management

1 Introduction

The recent years have witnessed an increasing development of new contracts, instruments, and solutions to transfer risk. These so called alternative risk transfer products are often provided by insurance and reinsurance companies and allow firms to deal with risk in a nontraditional way. One prominent example of such products are multiline (integrated risk management) policies that bundle different risk exposures to be covered by one insurance contract, with a common aggregate deductible and policy limit. The first contracts of this type combined property and casualty risks (see, e.g., Shimpi (2001)). "Twinpacks" that bundle only two—typically related—risks are still very common. But more and more multiline products have been developed to cover a whole array of different risks up to earnings per share insurance products, which essentially cover all major risks of a firm.

The virtue of theses programs is much debated. While some multiline products seem to be rather successful, others were failures and never implemented or dismantled after a few years. For that reason some practitioners are rather sceptical as to the virtue of multiline insurance and wonder whether it is a fad. To address this issue it is important to better understand the potential advantages of multiline products. Moreover, we have to better understand what risks one should combine and how the products should be designed (e.g., the level of aggregate deductible and policy limit). After all, some multiline products might have failed because of inappropriate design.

The literature often discusses multiline integrated risk management products as part of an enterprise-wide risk management strategy. Enterprise (or integrated) risk management implies that a firm's risks are viewed as a portfolio. Therefore, it seems rather natural to think about transferring or financing risks in terms of a portfolio as well, i.e., using a multiline product. But a portfolio view alone does not justify selling risks as portfolio. One rather has to carve out the difference between buying a portfolio of multiple insurance policies that each cover a single risk and buying a single insurance policy to cover a portfolio of multiple risks.

The typical reason in favour of multiline insurance discussed in the literature is that a common aggregate deductible on a portfolio of risks allows a reduction of total insurance coverage subject to some maximum aggregate risk that the firm is willing to retain (e.g., Shimpi, 2001, Harrington et al., 2002, and Meulbroek, 2002). This reduces the costs of the risk management program due to transaction costs (loading) of insurance.

Another important aspect of bundling risks in one insurance contract is the effect on the moral-hazard problem. In this paper I show that multiline policies allow for an improved trade-off between risk sharing and incentives. Consider a firm that is exposed to two identical and independent risks with two possible outcomes each, loss or no-loss. The firm can reduce the loss probability of each risk by investing in risk reduction (risk control). The investments are efficient but unobservable. Because of costs of financial distress or costs of ex post financing of losses, the firm wants to indemnify its losses through an insurance contract. But with complete insurance the firm has no incentives to invest in loss prevention.

I derive the optimal incentive-compatible insurance contracts, where the firm bears some risk in order to retain incentives to invest in risk control, for separate and joint insurance. The optimal retention structure depends on the underlying rationale for risk management, i.e., the firm's risk-management objective. I distinguish two stylized objectives, which are motivated in the paper: Subject to retaining incentives to implement the loss-prevention technologies for both risks, the firm minimizes (1) the maximum retained loss in any state or (2) the expected retained loss.

Imposing the additional constraint that the firm can hide, i.e., not report, losses, the optimal multiline contracts resemble standard insurance contracts with a joint deductible or policy limit. For a large set of "sufficiently low" loss probabilities, the joint insurance contract that minimizes the maximum retained loss resembles a multiline policy with a common aggregate deductible. The intuition is as follows: Using a multiline policy with a common aggregate deductible equal to the sum of the individual deductibles with separate insurance does not increase the maximum retained risk. But it does increase the loss that the firm has to bear if it incurs exactly one loss. For a large set of loss probabilities this increases incentives to invest in loss prevention, which allows to reduce the aggregate deductible, thereby reducing the maximum loss that the firm has to bear. But if the loss probability is very high (exceeding 50 percent), the additional retention in the state where the firm incurs exactly one loss actually reduces incentives to invest in loss reduction. Hence, it is optimal to have the firm participate in the loss only in the state where the firm incurs two losses. This is implemented through a common policy limit (and zero deductible).

A common aggregate policy limit is also optimal if the objective is to minimize the expected loss retained in the firm. The optimal contract is related to the one in an effort-incentive problem with a risk-neutral manager who is protected by limited liability. In order to minimize the expected wage payment the manager is paid a wage only in the state with the highest likelihood ratio. In the current setting the firm participates in the loss only in the state with the lowest likelihood ratio.

The paper is related to Fluet and Pannequin (1997) who analyze the role of bundling risks in multidimensional screening when risk-averse individuals have private information about the loss probabilities of the risks they face. The paper is also related to Laux (2001) who shows that combining projects to be managed by one manager reduces the costs of providing incentives in the presence of moral hazard.

The next section introduces the role of bundling to reduce transaction costs when insurance contracts are associated with loading. Section 3 discusses the role of bundling in the presence of moral-hazard problems when the firm has to retain risk to retain incentives to reduce the loss probabilities and wants to minimize (1) the maximum loss or (2) the expected loss it retains.

2 Reducing transaction costs of insurance

Assume that a firm has two identical and uncorrelated risks. Each risk can result in a loss L with probability p. The firm is risk neutral but there are costs associated with bearing losses, for example, costs of financing losses ex post, costs of financial distress, and costs of forgoing profitable investment opportunities. The firm can cover the losses by buying insurance. But insurance is associated with transaction costs (loading), which are assumed to be proportional to the expected insurance coverage. Therefore, it is not optimal for the firm to fully insure the risks.

Instead of explicitly modelling the trade-off between the transaction costs of insurance and the costs of bearing losses, I assume that there is a critical loss level $0 < \hat{L} < 2L$ as follows: incurring a loss that exceeds $\hat{L}$ results in very high costs of financial distress; the costs are zero for losses below $\hat{L}$. As a result, total retained losses must not exceed $\hat{L}$, but because of loading, it is optimal for the firm to retain losses up to $\hat{L}$. The optimal insurance contract minimizes the expected insurance coverage subject to retaining a maximum loss $\hat{L}$. This is equivalent to maximizing the expected retained loss subject to retaining a maximum loss $\hat{L}$ in any state.

If the two risks are insured individually, each contract $i \in \{a, b\}$ has to specify the level of insurance and retention of the underlying risk. For example, a deductible D_i is specified for each contract i. Maximizing the expected (total) retained risk subject to not exceeding $\hat{L}$, implies $D_a + D_b = \hat{L}$ and $D_i \leq L$ for $i \in \{a, b\}$. The expected loss to be covered by insurance is given by

$$p(L - D_a) + p(L - D_b) = p(2L - \hat{L}) \tag{1}$$

If the two risks are jointly insured, there is only one contract. The optimal deductible for this contract is $\hat{L}$ and the expected (total) loss to be covered by insurance is

$$p^2(2L - \hat{L}) + 2p(1 - p)\max\{0, L - \hat{L}\} \tag{2}$$

Comparing 1 and 2 directly yields that for every $\hat{L}$, with $0 < \hat{L} < 2L$, the expected loss to be covered by insurance is strictly lower if the risks are combined in one insurance contract than if different contracts are used. Equivalently, bundling the risks under one insurance contract with a deductible equal to the sum of the individual deductibles reduces the expected loss covered by insurance while holding the maximum loss retained by the firm constant.

The advantage of bundling risks is that the deductibles are used more "efficiently". If one contract indemnifies aggregate losses, the firm can always bear losses up to the critical level $\hat{L}$ and insure losses exceeding it. This is not possible if risks are insured individually. In this case the sum of the individual deductibles must not exceed $\hat{L}$. But $D_a + D_b = \hat{L}$ implies $D_i < \hat{L}$ for at least one insurance contract $i \in \{a, b\}$. As a consequence, when there is (exactly) one loss, the firm retains less risk than it is willing to bear. The firm is therefore overinsured in the sense that it buys more insurance coverage than it needs (e.g., Shimpi, 2001, Harrington et al., 2002, and Meulbroek, 2002).

The firm could reduce the insurance coverage with separate insurance by using conditional deductible levels in the individual contracts that increase to $\hat{L}$ if only one loss is realized. But this is exactly what is implemented by joint insurance.

3 Reducing Moral Hazard

3.1 Setting

The main change to the previous section is that the loss probabilities are no longer exogenous. Instead, the firm can reduce each risk's loss probability from p_h to p_l, with $1 > p_h > p_l > 0$, by investing c in loss prevention. The investment in loss prevention is efficient, even without costs of bearing losses, i.e., $p_l L + c < p_h L$, but unobservable.

If the firm fully insures both risks, it no longer invests in reducing the loss probabilities. Insurers anticipate the increase in the loss probability and the firm must bear the consequences in the form of a higher premium for the insurance policy. To retain incentives to make the investments, the firm must retain some of the loss.

Again, I assume that the firm is risk neutral but that there are costs associated with bearing losses. The cost of retaining a loss R is denoted by by $l(R)$. Losses exceeding the retention are insured. If there are transaction costs of insurance, the total premium that the firm has to pay for insurance exceeds the fair premium. The difference is the premium loading and denoted by $PL(.)$.

Optimization problem The firm's optimization problem is given by

$$\min_{\mathbf{R},\phi} E[L|\phi] + E[l(R)|\mathbf{R},\phi] + \phi c + PL(\mathbf{R},\phi) \tag{3}$$

s.t.

$$E[R + l(R)|\mathbf{R}, \phi] + \phi c \geq E[R + l(R)|\mathbf{R}, \phi'] + \phi' c \text{ for all } \phi' \neq \phi, \tag{4}$$

$$R_a, R_b, R_1, R_2 \geq 0 \tag{5}$$

with $\mathbf{R} = (R_a, R_b)$ or (R_1, R_2) and $\phi \in \{0, 1, 2\}$. $\mathbf{R}$ indicates the choice of contract: If the firm chooses separate contracts, R_a and R_b are the retention levels in contracts a and b respectively; if the firm chooses joint insurance, R_1 and R_2 are the retention levels for total losses of L and $2L$ respectively. ϕ indicates the investment in loss prevention; if $\phi = i$, the firm invests in loss prevention for i risks.

The firm chooses $\mathbf{R}$ and ϕ to minimize the sum of the expected total loss, $E[L|\phi]$, the expected costs of retaining risk, $E[l(R)|\mathbf{R}, \phi]$, the costs of reducing risk, ϕc, and the costs of transferring risk, $PL(\mathbf{R}, \phi)$. $\mathbf{R}$ and ϕ have to satisfy the firm's incentive compatibility constraint (4).

To focus on moral hazard, I assume that insurance contracts are offered in a competitive market at a fair premium. That is, premium loading is zero and $PL(\mathbf{R}, \phi) = 0$.First best then implies zero retention and $\phi = 2$.

As the investment in loss prevention is unobservable, first best is not implementable because it is optimal for the firm not to invest in loss prevention when it does not retain any risk.

The optimization problem can be solved in two steps. First, for every $\phi \in \{0, 1, 2\}$ the firm minimizes the retention subject to (4). Second, the firm chooses the combination of ϕ and retention that minimizes (4).

For $\phi = 0$, the optimal retention is zero; there is no role for joint insurance. For $\phi = 1$, it is optimal to choose zero retention for one loss and there is again no role for joint insurance.

Therefore, the interesting case is $\phi = 2$. In this case the firm can choose between separate and joint insurance.

In what follows I analyzes whether and how it is possible to reduce the expected loss of retaining risk, subject to the incentive compatibility constraint of investing in loss prevention for both risks, by choosing joint insurance. The optimal retention structure depends on the loss function $l(R)$. I consider two extremes.

(1) As in the previous section there is a critical loss level $\hat{L}$:

$$l(R) = \begin{cases} 0 & \text{if } R \leq \hat{L} \\ \infty & \text{if } R > \hat{L} \end{cases}$$

(2) The cost of retaining risk are linear:

$$l(R) = kR$$

For loss function (1), the question is whether joint insurance allows to reduce the maximum (total) retained loss (below $\hat{L}$). I capture this by assuming that the objective is to minimize the maximum retention. (Of course it is not optimal for the firm to choose $\phi = 2$ if the necessary retention exceeds $\hat{L}$.)

For (2) the objective is to minimize the expected (total) retained loss. The objective of minimizing the expected retained loss arises if the costs of retaining a loss are proportional to the retained loss. For example, the cost of ex post financing a loss with external funds may be proportional in the amount of funds raised.

Of course, the cost functions that are underlying the two objectives are rather stylized. But they serve two purposes: First, they show how sensitive the optimal structure of multiline insurance products is to the firm's objective. Hence, when designing integrated risk policies, one has to be rather careful in carving out the potential reasons for why the firm wants to manage its risks. Second, the two examples illustrate the potential advantage of multiline insurance products.

3.2 Separate contracts

If each risk is covered by an individual insurance policy, the retention levels R_a and R_b have to satisfy the firm's incentive constraints for the individual risks[1]

$$\begin{aligned} p_l R_a + c &\leq p_h R_a, \\ p_l R_b + c &\leq p_h R_b. \end{aligned}$$

Because of symmetry $R_a^* = R_b^* \equiv R^*$. The optimal retention level R^* for each contract, which minimizes (1) the maximum retained loss *and* (2) the expected retained loss, is given by[2]

$$R^* = \frac{c}{p_h - p_l}.$$

3.3 Joint contract: Indemnifying total losses

If both risks are jointly covered by a single insurance policy, the contract has to specify the retention levels in the two possible loss states, L (one loss) and $2L$ (two losses). They are denoted by R_1 and R_2 respectively. State 1 (one loss) occurs with probability $p_i(1 -$

[1] Without loss of generality I ignore the losses of bearing risk in the formal analysis and focus only on the retained risk.

[2] (1) and (2) yield identical retention structures because there is only one loss-state for each risk. In a more general setting, with multiple loss states for every risk, the retention structures that are optimal for (1) and (2) generally differ. The qualitative results of the paper, however, do not depend on the two-state distribution.

$p_j) + p_j(1 - p_i)$ and state 2 (two losses) occurs with probability $p_i p_j$, where $p_i \in \{p_h, p_l\}$ and $p_j \in \{p_h, p_l\}$ are determined by the firm's investments in risk prevention. The firm's incentive constraints are given by

$$p_l^2 R_2 + 2p_l(1 - p_l)R_1 + 2c \leq p_h p_l R_2 + (p_h(1 - p_l) + p_l(1 - p_h))R_1 + c \quad \text{(IC1)}$$

$$p_l^2 R_2 + 2p_l(1 - p_l)R_1 + 2c \leq p_h^2 R_2 + 2p_h(1 - p_h)R_1. \quad \text{(IC2)}$$

IC1 assures that the firm does not shirk on one of the risks and IC2 assures that the firm does not shirk on both risks.[3] The optimal retention levels R_1 and R_2 depend on the firm's objective, i.e., whether the firm minimizes (1) the maximum aggregate retained loss or (2) the expected retained loss.

(1) Minimizing the maximum retained loss. I first analyze whether joint insurance allows to reduce the retained loss. The optimal insurance contract is characterized in the following Lemma:

Lemma 1: *The incentive compatible insurance contract that minimizes the maximum retained loss, i.e.,* $\min \max\{R_1, R_2\}$, *has the following properties:*

(a) If $p_h + p_l < 1$,

$$R_1^* = R_2^* = \frac{2c}{2(p_h - p_l) - (p_h^2 - p_l^2)}$$

(b) If $p_l > 0.5$,

$$R_1^* = 0, \ R_2^* = \frac{c}{p_l(p_h - p_l)}$$

[3] Because of symmetry I do not have to distinguish between the two individual risks; IC2 captures both risks in the sense that (i) if the firm has an incentive to shirk on (exactly) one risk, it is indifferent between the two, and (ii) if it has no incentive to shirk on risk $i \in \{a, b\}$, it has no incentive to shirk on $j \neq i$.

(c) If $p_h + p_l \geq 1$ *and* $p_l \leq 0.5$*,*

$$R_1^* = \frac{c}{p_h - p_l}, \; R_2^* = 2R_1^*$$

Proof: Rearranging incentive constraints 1 and 2 yields

$$c \leq (p_h p_l - p_l^2)R_2 + (p_h - p_l)(1 - 2p_l)R_1 \quad \text{(IC1)}$$

$$2c \leq (p_h^2 - p_l^2)R_2 + 2(p_h - p_l)(1 - p_h - p_l)R_1. \quad \text{(IC2)}$$

Both incentive constraints are binding if one replicates the retention structure of separate contracts, i.e., $R_2 = 2R^*$ and $R_1 = R^*$ with $R^* = \frac{c}{p_h - p_l}$. For this contract, $R_2 > R_1$. Minimizing the maximum retention therefore implies reducing R_2 without violating the incentive constraints. Clearly this is possible if and only if both incentive constraints can be relaxed by changing R_1 (i.e., the right hand side of both constraints increases), which is possible if and only if either (a) $p_h + p_l < 1$ or (b) $p_l > 0.5$.

(a) $(p_h - p_l)(1 - 2p_l) > 0$ *and* $2(p_h - p_l)(1 - p_h - p_l) > 0 \Leftrightarrow p_h + p_l < 1$.[4] In this case, increasing R_1 relaxes both constraints. It is therefore possible to reduce R_2 without violating the incentive constraint by simultaneously increasing R_1. $\min \max\{R_1, R_2\}$ implies $R_1 = R_2$ subject to IC1 and IC2. It is straightforward to check that for $R_1 = R_2$, IC1 is implied by IC2 and $R_2^* = R_1^* = \frac{2c}{2(p_h - p_l) - (p_h^2 - p_l^2)}$.

(b) $(p_h - p_l)(1 - 2p_l) < 0$ *and* $2(p_h - p_l)(1 - p_h - p_l) < 0 \Leftrightarrow p_l > 0.5$.[5] Reducing R_1 now relaxes both constraints. It is therefore possible to reduce R_2 without violating the incentive

[4] $2(p_h - p_l)(1 - p_h - p_l) > 0 \Leftrightarrow p_h + p_l < 1$ and $(p_h - p_l)(1 - 2p_l) > 0 \Leftrightarrow p_l < 0.5$. In addition, $p_h + p_l < 1 \Rightarrow p_l < 0.5$.

[5] $2(p_h - p_l)(1 - p_h - p_l) < 0 \Leftrightarrow p_h + p_l > 1$ and $(p_h - p_l)(1 - 2p_l) < 0 \Leftrightarrow p_l > 0.5$. In addition, $p_l > 0.5 \Rightarrow p_h + p_l > 1$.

constraints by simultaneously decreasing R_1. $\min\max\{R_1, R_2\}$ and $R_1 \geq 0$ implies $R_1^* = 0$. For $R_1^* = 0$, IC2 is implied by IC1 and $R_2^* = \frac{c}{p_l(p_h - p_l)}$.

(c) If $p_h + p_l \geq 1$ and $0.5 \leq p_l$ it is not possible to relax *both* constraints by changing R_1 and it is therefore not possible to reduce R_2 without violating at least one of the incentive constraints. Hence, $R_2^* = 2R_1^* = \frac{2c}{p_h - p_l}$ is optimal.[6] *Q.E.D.*

In cases (a) and (b) it is optimal for the firm to deviate from the retention structure implied by separate insurance, which is chosen in case (c). In case (a) the probability of incurring exactly one loss is higher if the firm shirks (only one or no investment in loss prevention) than if it invests in loss prevention for both risks. Therefore, retaining a loss if the firm's total loss is L has a positive incentive effect. Thus, increasing R_1 increases incentives and allows to reduce R_2 without violating the incentive constraints. Minimizing the maximum retention in any state implies that the retention is the same in both states. Case (a) captures a reasonable situation where the possible loss probabilities are not too high, satisfying $p_h + p_l < 1$. In case (b), in contrast, the probability of a loss is very high, with $p_l > 0.5$. Even though one may question the practical relevance of this case, it is nevertheless illustrative to consider the implications for the optimal insurance policy. For $p_l > 0.5$ the probability of incurring exactly one loss, L, is actually *higher* if the firm does not shirk. Hence, the incentive effect of retaining a loss in state 1 is negative and it is optimal to reduce the retained risk in this state to zero. This allows to reduce R_2.

The following proposition directly follows from the discussion of the possibility to reduce the total retention, R_2, when combining risks.

---

[6]For $p_l = 0.5$, $R_1 = 0$ and $R_2 = \frac{c}{p_l(p_h - p_l)}$ also satisfies both incentive constraints. The firm is indifferent between this contract and $R_2 = 2R_1 = \frac{2c}{p_h - p_l}$ since $\min\max\{R_1, R_2\} = \frac{c}{p_l(p_h - p_l)} = \frac{2c}{p_h - p_l}$ for $p_l = 0.5$.

Proposition 1: *If (a) $p_h + p_l < 1$ or (b) $p_l > 0.5$, the maximum loss that the firm has to bear in the optimal incentive compatible insurance contract is strictly lower for joint insurance than for separate insurance.*

Given the optimality of multiline insurance in cases (a) and (b) it is interesting to analyze the optimal contract or retention structure in more detail. In case (a) the advantage stems from using the retention levels in states 1 and 2 more evenly. This is not possible if the two risks are indemnified individually. In this case the individual retentions, which are necessary to provide incentives for each individual risk, add up if the firm incurs two losses. With joint insurance, the firm can increase the retention in the one-loss state, thereby increasing incentives to reduce the loss probability of both risks. In case (b), in contrast, the advantage stems from using the retention in less states. As the likelihood of incurring exactly one loss increases when the firm invests in loss prevention, the incentive constraints are relaxed if the retention in state 1 is reduced. In this case, $R_1 \geq 0$ is binding.[7] $R_1 \geq 0$ implies that there is no overinsurance and is justified to prevent fraudulent losses.

Two additional constraints have to be imposed if the firm is able to hide, i.e., not report, losses. These constraints are akin to the constraints imposed by Innes (1990) on financial contracts. First, the insurance coverage must be nondecreasing, i.e., the marginal increase in retention must not exceed the marginal increase in loss. Second, there is no additional punishment for losses, i.e., the retention in any state must not exceed the loss in this state.

[7] It would be optimal to overinsure in state 1, i.e., the firm gets a reward if the aggreate loss is L. Reducing R_1 below zero (i.e., paying a reward for state 1) allows to further reduce R_2. If R_1 is sufficiently low (i.e., the reward is sufficiently high), $R_2 = 0$ and the firm retains incentives without retaining any risks. Punishments (retained losses) are substituted by rewards. The requirement that there is no overinsurance is often justified by arguing that otherwise the firm actually has an incentive to incur the loss.

In the present setting both constraints imply (i) $R_1 \leq L$ and (ii) $R_2 \leq R_1 + L$.[8]

(i) may be binding in case (a), while (ii) may be binding in case (b). Imposing these constraints can affect the optimal structure of a multiline insurance policy. However, Proposition 1 still holds. In case (a), if $R_1 \leq L$ is binding, the optimal contract is $R_1^* = L$ and $R_2^* = 2L - 2\frac{c-(p_h+p_l)L}{p_h^2-p_l^2}$.[9] If $R_2 \leq R_1 + L$ is binding in case (b), the optimal contract is $R_2^* = R_1^* + L$ and $R_1^* = \frac{c}{(1-p_l)p_h-p_l} - \frac{p_l}{1-p_l}L$.

Proposition 2: *Given constraints (i) $R_1 \leq L$ and (ii) $R_2 \leq R_1 + L$, the optimal incentive compatible multiline insurance policy that minimizes the maximum retained loss can be implemented as a standard policy with an aggregate deductible D or an aggregate policy limit I.*

(a) If $p_h + p_l < 1$, the firm bears aggregate losses up to D while aggregate losses exceeding D are covered by insurance, with

$$D = \begin{cases} \frac{2c}{2(p_h-p_l)-(p_h^2-p_l^2)} \leq L & \text{if } R_1 \leq L \text{ is not binding} \\ 2L - 2\frac{c-(p_h+p_l)L}{p_h^2-p_l^2} > L & \text{if } R_1 \leq L \text{ is binding} \end{cases}$$

(b) If $p_l > 0.5$, the firm indemnifies aggregate losses up to I and retains losses exceeding I, with

$$I = \begin{cases} 2L - \frac{c}{p_l(p_h-p_l)} \geq L & \text{if } R_2 \leq R_1 + L \text{ is not binding} \\ \frac{c}{(1-p_l)p_h-p_l} - \frac{p_l}{1-p_l}L < L & \text{if } R_2 \leq R_1 + L \text{ is binding} \end{cases}$$

---

[8] (i) and (ii) imply $R_2 \leq 2L$.

[9] Assume $R_2^* = R_1^* > L$ in Lemma 1(a). This contract is not possible with the constraint $R_1 \leq L$. In this case the maximum retention in the one-loss state is L. From the discussion of the optimal contract in Lemma 1, reducing R_1 from R_1^* to L necessitates an increase of R_2 above R_2^* to satisfy IC2, which is binding for $R_2^* = R_1^*$. Given $R_1 = L$, the constraint is binding for $R_2 = [2c - 2(p_h - p_l)(1 - p_h - p_l)L]/(p_h^2 - p_l^2) = 2L - 2[c - (p_h + p_l)L]/(p_h^2 - p_l^2)$.

(2) Minimizing the expected retained loss. A different insurance contract is optimal if the firm's objective is to minimize the expected retained loss.

Lemma 2: *The following retention structure provides the firm with incentives to invest in risk reduction for both risks while minimizing the expected retained exposure:*

$$R_1^* = 0,\ R_2^* = \frac{c}{p_h p_l - p_l^2}.$$

Proof. The first step of the proof is to show that any retention structure is dominated by one where the firm participates in the loss if and only if it incurs two losses (i.e., in state 2). Let $(\overline{R}_1, \overline{R}_2)$ be an incentive compatible retention structure with $\overline{R}_1 > 0$. Replacing this retention structure by $\widehat{R}_1 = 0$ and $\widehat{R}_2 = \overline{R}_2 + 2\frac{1-p_l}{p_l}\overline{R}_1$ yields the same expected loss borne in equilibrium and relaxes both incentive constraints. That is, the right hand side of both constraints is strictly higher for $(0, \widehat{R}_2)$ than for $(\overline{R}_1, \overline{R}_2)$:[10]

$$\begin{aligned} p_h p_l \overline{R}_2 + (p_h(1-p_l) + p_l(1-p_h))\overline{R}_1 &< p_h p_l \widehat{R}_2 \\ p_h^2 \overline{R}_2 + 2p_h(1-p_h)\overline{R}_1 &< p_h^2 \widehat{R}_2. \end{aligned}$$

Substituting for $\widehat{R}_2$ and rearranging terms, yields $p_l < p_h$ for both conditions, which are therefore always satisfied. Hence, it is optimal to choose $R_1 = 0$ and $R_2 > 0$. The next step is to derive the optimal retention R_2, which is the minimum retention that satisfies constraints 1 and 2. For $R_2^* = \frac{c}{p_h p_l - p_l^2}$, IC1 is binding and IC2 is satisfied. *Q.E.D.*

Proposition 3: *The minimum incentive-compatible expected retention is strictly lower with a multiline insurance contract that indemnifies aggregate losses than if each risk is indemnified by an individual contract.*

[10] The left hand sides of the constraints are the same for $(\overline{R}_1, \overline{R}_2)$ and $(0, \widehat{R}_2)$ because $(0, \widehat{R}_2)$ is chosen to have the same expected loss in equilibrium as $(\overline{R}_1, \overline{R}_2)$.

Proof. Given Lemma 2 and the optimal retention with separate indemnification, the proposition directly follows from comparing total expected losses. It is straightforward to see that $p_l^2 \frac{c}{p_h p_l - p_l^2} < 2p_l \frac{c}{p_h - p_l}$. *Q.E.D.*

A multiline insurance product can be structured to reduce the expected loss that the firm has to bear to retain incentives to implement the loss-prevention investments. The result is related to Laux (2001) where it is shown that bundling projects to be managed by a single manager reduces expected wage costs of providing incentives to exert high effort on all projects when the manager is protected by limited liability. The setting is similar to the one here. In Laux (2001) the manager is rewarded if and only if all projects are successful in order to provide the manager with incentives to increase the probability of a success for each project.[11] Here the firm bears part of the loss if and only if all risks realize a loss in order to induce the firm to reduce the probability of a loss for each risk. One difference is noteworthy. If the incentives are to be provided to increase the success-probabilities of multiple projects, the binding constraint is that the manager has no incentive to shirk on all projects: the probability of obtaining a reward decreases when shirking on one project and therefore the cost of shirking on an additional project decreases. In contrast, if incentives are to be provided to reduce the loss-probabilities of multiple risks, the relevant constraint is that the manager must not shirk on one risk: the probability of bearing a loss increases after shirking on one project and therefore the cost of shirking on an additional project increases. This difference has important implications for the robustness of the results. If the manager

[11] Laux (2001) cosidered the general case of N projects. The discussion in the present paper can also be extended to N projects with a joint retention if and if all risks incur a loss beeing optimal and the binding constriant that the firm has no incentive to shirk on one risk. Laux (2001) also shows that the advantage of combining projects extends to multiple states.

receives a wage only if all projects are successful, the manager stops exerting effort if he can observe (at an interim stage) that one of the projects failed. This is not the case with the insurance contract. Observing that one of the risks incurs a loss tightens incentives to invest in loss prevention for the other risks.

Imposing the constraint $R_2 \leq R_1 + L$ can change the optimal contract without affecting Proposition 3. If the constraint binds, $R_2^* = R_1^* + L$ and $R_1^* = c/(1-p_l)(p_h - p_l) - p_l L/(1-p_l)$.

Proposition 4: *Given the constraint $R_2 \leq R_1 + L$, the optimal incentive compatible multiline insurance contract that minimizes the expected retained loss can be implemented as a standard policy with an aggregate policy limit I, given by*

$$I = \begin{cases} 2L - \frac{c}{p_h p_l - p_l^2} \geq L & \text{if } R_2 \leq R_1 + L \text{ is not binding} \\ \frac{L}{1-p_l} - \frac{c}{(1-p_l)(p_h - p_l)} < L & \text{if } R_2 \leq R_1 + L \text{ is binding} \end{cases}$$

References

[1] Fluet, Claude, and Francois Pannequin: Complete versus incomplete insurance contracts under adverse selection with multiple risks, *Geneva Papers on Risk and Insurance Theory* 22, 1997, 82-101.

[2] Harrington, Scott E., and Greg Niehaus, and Kenneth J. Risko: Enterprise Risk Management: The case of United Grain Growers, *Journal of Applied Corporate Finance* 14, 2002, 71-81.

[3] Innes, R.D.: Limited liability and incentive contracting with ex-ante action choices, *Journal of Economic Theory* 52, 1990, 45-67.

[4] Laux, Christian: Limited liability and incentive contracting with multiple projects, *RAND Journal of Economics* 32, 2001, 514-526.

[5] Meulbroek, Lisa: The Promise and Challenge of Integrated Risk Management, *Risk Management and Insurance Review* 5, 2002, 56-70.

[6] Shimpi, Prakash: Multi-line and multi-trigger products, in: *Integrating corporate risk management*, P. Shimpi (Ed.), Texere, New York and London, 1999/2001, 101-124.